Design and analysis of machine components

Follow forces and motion through a component, derive its limiting response and compare a design estimate with an observable test of the assembled machine.

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A fatigue limit measured on a polished specimen does not directly describe a shoulder on a production shaft. Track where each correction enters and how the measured torque or bending history changes the inferred life.

7. Will tooth-root bending or flank contact govern a gear pair?

The same torque loads the tooth root and the mating flanks. Calculate two distinct demands, carry the same duty blocks through both, and see why doubling the face width gives different gains.

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Two transmissions can deliver the same speed reduction around a right angle yet require very different bearings and cooling. Derive the directions of the loads and the dependence of efficiency on lead angle.

9. Can a mating pin enter every accepted hole?

A rectangular coordinate tolerance and a circular position tolerance accept different parts. Carry a hole requirement from its datum reference frame through its material condition to an actual inspection decision.

Design and analysis of machine components · 1. Cam displacement laws, contact geometry and follower acceleration

An inspection station uses a roller to lift a specimen for imaging, hold it and return it. Specifying a ten-millimetre rise determines neither the force on the guide nor whether the follower can follow the prescribed geometry. Construct two different motions through the same endpoints and separate their geometric and dynamic consequences.

Construct the pitch curve from the prescribed motion

Imagine holding the cam still and rotating the entire guide around it in the opposite direction. The centre of an inline roller traces a polar curve in the cam frame. Let the prime radius be the distance from cam axis to roller centre at zero lift. At angle θ the pitch radius is $r = R + s(\theta)$. A small rotation advances the centre tangentially by $r\,d\theta$ and radially by $s'\,d\theta$. The tangent therefore makes an angle with the circumferential direction whose tangent is the ratio of these increments. The common normal makes the same angle with the radial guide direction. For a frictionless, inline translating roller the pressure angle follows immediately:

$$\tan\alpha = \frac{s'(\theta)}{R + s(\theta)},$$

$$N\cos\alpha = F,$$

$$F_{\text{guide}} = F\tan\alpha.$$

Here the force along the follower includes inertia and any spring or specimen load. Reversing the rise changes the sign of the lateral reaction. Angular derivatives must be per radian because the tangential increment is an arc length. A calculation performed per degree systematically underestimates the slope by a factor of about fifty-seven. The actual metal profile is a normal offset of the pitch curve by the roller radius. Simply subtracting the roller radius radially works on a concentric dwell but generally gives the wrong point on the rise.

Compare motion laws at a dwell boundary

Write $u = \theta/\beta$ from zero to one on the rise. A harmonic motion is half a cosine. A cycloidal motion adds a sinusoidal correction to a linear ramp. Both end at zero velocity, making a stationary dwell possible without a jump in speed. Differentiation resolves what else happens:

$$s_H = \tfrac{h}{2}(1 - \cos\pi u),$$

$$s_C = h\left(u - \frac{\sin 2\pi u}{2\pi}\right),$$

$$s_C' = \frac{h}{\beta}(1 - \cos 2\pi u).$$

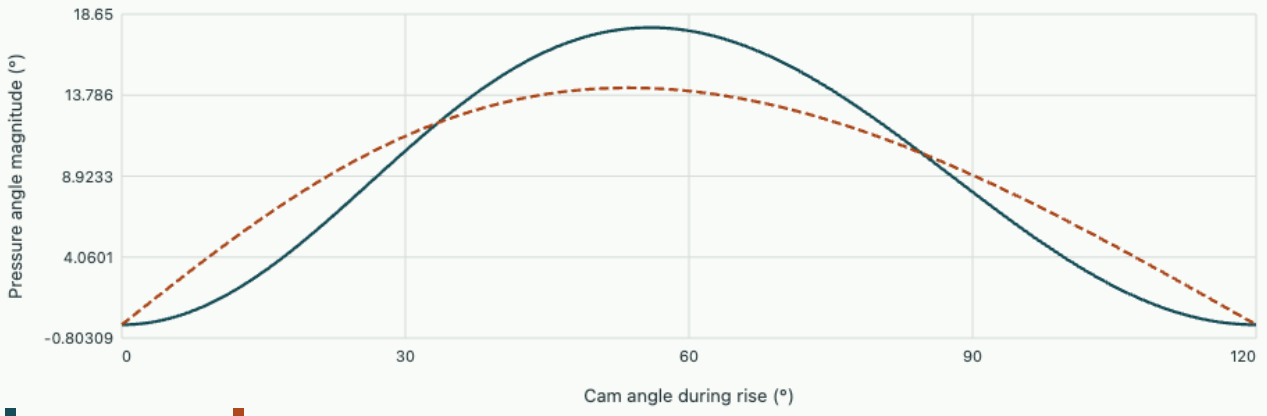
For constant angular speed the acceleration is $\Omega^2 s''$. Harmonic acceleration has a finite nonzero value just inside the rise and vanishes on the dwell. The cycloidal law has zero endpoint acceleration, although its jerk changes when entering the dwell. At mid-rise the cycloidal slope is greater. Thus an improvement in acceleration continuity can cost additional side load. At a varying spindle speed add $s'\dot{\Omega}$ by the chain rule. A measured acceleration trace during run-up can disagree with a constant-speed prediction even with an accurately cut contour.

For 10 mm rise through 120 degrees from a prime radius of 25 mm, the middle cycloidal slope is 9.55 mm per radian. The pitch radius there is 30 mm and the pressure angle is 17.66 degrees. The slope reaches its maximum there, yet the largest angle occurs slightly earlier because the denominator was still increasing. Compare both laws across the complete rise. Increase only the prime radius and predict whether the slope or the guide force changes.

Open the live experiment

Try the calculation

Lift (mm)	**10**
Rise angle (°)	**120**
Prime radius (mm)	**25**



The cycloidal peak pressure angle is 17.85°. At half rise it is 17.66°; the harmonic comparison has an acceleration jump at a dwell.

Background · How does changing the cam speed affect acceleration and jerk?

The prescribed lift $s(\theta)$ is a geometric function. At constant cam angular speed Ω, differentiating with respect to time multiplies each angular derivative by another power of speed. Angles in these derivatives are expressed in radians.

$$v = \Omega s'(\theta),$$

$$a = \Omega^2 s''(\theta),$$

$$j = \Omega^3 s'''(\theta).$$

Doubling the speed leaves the traced pitch curve and peak lift unchanged while doubling peak follower speed, quadrupling peak acceleration and multiplying peak jerk by eight. The inertial contribution to the normal contact load can therefore reach the available preload at a speed where the slow test runs quietly. During spin-up the acceleration also includes $s'\dot{\Omega}$.

How a smooth pitch curve can make a poor physical cam

The curvature depends on the second derivative as well as the pressure angle. For a polar curve, differentiate its Cartesian position twice and apply the signed planar curvature formula. With outward radial coordinates its signed radius, positive on a circular convex arc, is

$$\rho_p = \frac{(r^2 + r'^2)^{3/2}}{r^2 + 2r'^2 - rr''}.$$

On a convex segment the inward offset by roller radius has signed radius $\rho_p - r_{\mathrm{roller}}$. At zero an offset develops a cusp. A smaller roller helps in this local geometric check but can raise contact pressure at fixed normal force. Numerical differentiation of a measured cam profile twice amplifies encoder noise; estimate curvature on a documented angular window and repeat the calculation with another window. A sharp reproducible feature demands a different explanation from a peak moving with the smoothing window.

Reconstruct the motion from a camera and phase mark

Place an angular reference on the shaft and record the follower at equally spaced phase values for several turns. Average positions at equal phase to reduce uncorrelated readout noise. Differentiate the average only after expressing the phase in radians. A centred local fit yields slope and curvature with more usable precision than subtracting adjacent noisy samples twice. Inspect the first and last part of the rise separately, because smoothing across a dwell hides a real acceleration jump in the harmonic schedule.

As a cross-check, integrate the estimated derivative over the rise. It should recover the measured ten-millimetre travel. At the same time inspect the pressure angle derived from it. A fit can preserve the correct total travel while underestimating the peak local slope through excessive smoothing. Independent displacement and guide-force measurements at several phase angles help distinguish these two errors.

Tutorial · Turn a lift diagram into a measured acceleration

A cycloidal 10 mm lift takes 120 degrees at 300 rpm. Find the peak follower acceleration and the value after doubling the speed with the same cam.

▼ **Convert the time base before differentiating**

The rise lasts one third of a 0.200 s revolution, giving 0.06667 s. Differentiating the displacement twice in physical time gives peak acceleration $2\pi h/t_r^2 = 14.14$ metres per second squared. At 600 rpm the rise time halves and peak acceleration reaches 56.55 metres per second squared. The corresponding inertial contribution of a moving mass of 0.20 kg grows from 2.83 to 11.31 N. It changes sign across the rise. A spring holding the roller in contact consequently has a different margin on the two halves.

Tutorial · Interpret a guide measurement

A force transducer reads 120 N along the guide at the middle of the slower rise. Estimate contact and lateral force. Would a larger prime radius halve the axial force?

▼ **Resolve the normal with the pitch-curve slope**

Using the angle above gives a lateral force of 38.2 N and normal load of approximately 126 N. Increasing radius reduces the angle at the same kinematic point. The specified axial force comes from specimen, spring and moving mass. Reducing angle changes its resolution into normal and transverse components, while the prescribed lift history still requests the same axial acceleration. Guide friction couples the directions and would require a fresh force balance. A changing spring lever arm could change the applied axial force as well.

Tutorial · Check the roller offset independently

A convex portion of a pitch curve has local radius 8 mm; the proposed roller radius is 9 mm. The maximum pressure angle passes the design team's geometric requirement. Explain whether the nominal profile can realise that local curvature.

▼ **Inspect the normal offset**

The local offset radius would be minus one millimetre after crossing a cusp at eight. The normal-offset construction folds and the roller cannot follow the intended envelope there. Pressure angle uses the first derivative, so it cannot certify curvature. Test a smaller roller or a wider rise and recalculate normal load and contact stress. A tolerance on the measured first derivative alone leaves this failure undetected. Recheck the return as well, because its angular duration and signed curvature can differ.

Carnegie Mellon · Introduction to Mechanisms, chapter 6, cam motion and profile inversion

Design and analysis of machine components · 2. Helical gears, mesh forces and bearing reactions

A motor supplies a known torque to a helical pinion. Select the free body carefully: torque fixes one component of the tooth load, the helix creates thrust, and the location of that thrust can alter bearing reactions even in a parallel-shaft transmission. Build the force diagram and reverse it from measured support loads.

Unroll a pitch cylinder and resolve the tooth normal

Imagine cutting the pitch cylinder and unwrapping it. The tooth trace makes helix angle β with the shaft axis. The component of tooth contact force lying in the unwrapped pitch surface points perpendicular to that trace. It has circumferential projection F_t and axial projection F_a. The line of action also tilts out of the tangent plane through normal pressure angle α_n, creating a radial component. For a frictionless external mesh the magnitudes follow from two right triangles and torque balance:

$$F_t = \frac{2T}{d},$$

$$F_a = F_t \tan\beta,$$

$$F_r = \frac{F_t \tan\alpha_n}{\cos\beta},$$

$$F_n = \frac{F_t}{\cos\beta\cos\alpha_n}.$$

Projection of the transverse circular tooth pitch onto its normal section gives $p_n = p_t \cos\beta$, so the pitch diameter of a standard unshifted gear is $d = z m_n / \cos\beta$. Distinguish fixed diameter from fixed tooth count and normal module when changing helix angle. At fixed diameter and torque the tangential force stays fixed; changing the diameter changes it too. A spur gear emerges continuously as the helix angle tends to zero, because axial thrust vanishes and both pressure angles coincide.

For 100 N m, 100 mm reference diameter and 20 degrees for both helix and normal pressure angles, the tangential, axial and radial forces are respectively 2000, 728 and 775 N. Their vector magnitude is 2265 N. The direction of thrust depends on helix hand and torque direction. The two mating gears experience equal and opposite contact forces in common coordinates. Two separate diagrams drawn from opposite shaft ends can make consistent thrust arrows look identical on the paper.

Transport the mesh wrench to the shaft centreline

Let bearing A lie at coordinate zero and B at span L. A lateral force acts a distance a from A. In each independent bending plane its reaction magnitudes for a simple support model are $F(L-a)/L$ and Fa/L, with directions opposite the applied force. The axial force acts at the pitch contact, a radius away from the centreline; moving it onto the shaft centre adds a couple of magnitude $F_a d/2$. In a plane where the radius points vertically this couple acts in the vertical bending plane. Determine its sign from the cross product. The tangential force at that radius also contributes the transmitted torsional moment, balanced by the drive.

For $a = 60$ mm, $L = 150$ mm, the 2000 N tangential force gives 1200 and 800 N. Ignoring the thrust couple, the radial force would give 465 and 310 N. The 36.4 N m couple transfers 243 N between the two radial reactions without changing their sum. With the sign increasing B, the radial reactions become approximately 222 and 553 N. Dynamic load ratings use the resultant load on each bearing and a catalogue rule for its combination with axial force. Suppose A locates the shaft axially and B permits axial displacement. Then A reacts the full thrust in this idealisation. Two preloaded locating bearings require deformation compatibility, clearance and stiffness for a reliable split.

Measure a signature of mesh geometry

Make two steady tests at the same torque magnitude, reversing rotation and applied torque together. The separating radial component of an external mesh usually stays separating while thrust changes direction with the loaded flank. Tangential reactions reverse sign. Resolve measured reactions into a common coordinate frame and subtract bearing preload before comparing. A change in torque sign may also move load to a different bearing shoulder or traverse backlash. Examine displacement on each side of the reversal before attributing changes to helix geometry.

Use the involute-contact study to distinguish tooth contact ratio from the total steady mesh force.

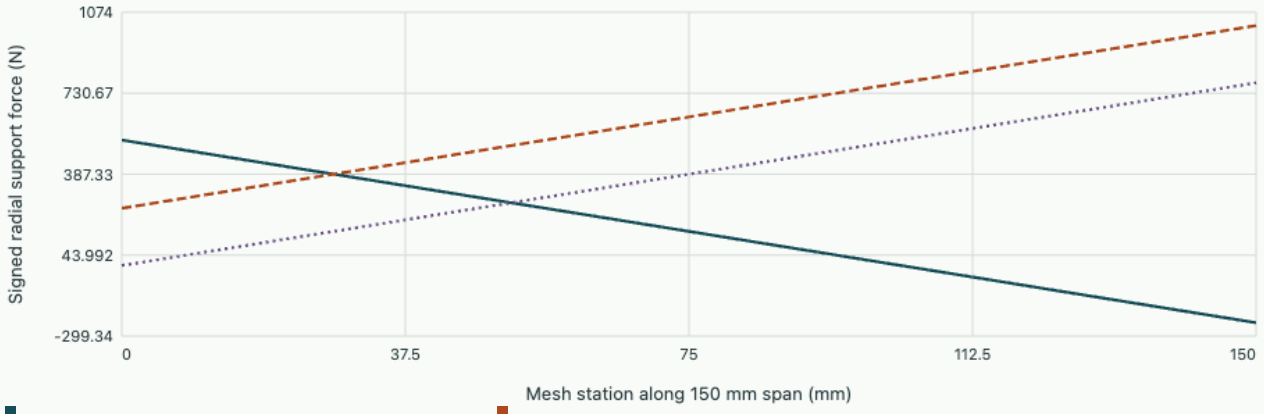
Move the mesh along the shaft without changing the mesh forces

The experiment holds torque and pitch diameter fixed while shifting the mesh station. In each radial reaction the contribution of the lateral force varies linearly with station. The off-axis thrust contribution is instead a constant signed transfer between the supports for this geometry. Compare the two curves for support B. Their vertical separation is the thrust couple divided by span and stays fixed as the gear moves. The sum of A and B stays equal to the separating force at every station.

Open the live experiment

Try the calculation

Shaft torque (N m)	100
Pitch diameter (mm)	100
Helix angle (°)	20



For the marked 60 mm station, radial reactions are 222.1 and 552.5 N. The 727.9 N thrust acts at pitch radius and supplies a 36.40 N m couple.

Near one end the smaller support reaction can change sign. In the signed vector convention this means the support must react in the opposite direction. A preloaded bearing can change its incremental force in this direction. A clearance support might instead unload until the opposite side takes up load. The simple-support drawing chooses neither actual contact path. Check axial location, allowable movement and measured preload before interpreting a predicted negative incremental load as a reversed material stress at one rolling contact.

Tutorial · Size from the normal module

An unshifted pinion has 30 teeth, normal module 3 mm and helix angle 25 degrees. At 90 N m find pitch diameter, tangential load and axial thrust.

▼ **Resolve geometry before applying torque**

The transverse module is $3/\cos 25^\circ = 3.31$ mm, giving diameter 99.3 mm. Consequently the tangential force is 1813 N and axial thrust 845 N. Using 90 mm as diameter would overestimate transmitted force. The torque check multiplies 1813 N by a 49.65 mm arm, recovering approximately 90 N m. The input power cannot be recovered without the rotation speed; steady torque does not by itself determine heating or time to a given number of bearing revolutions.

Tutorial · Find a missing transverse couple

Radial support reactions sum to 780 N in a 150 mm span. The mesh lies 60 mm from A, and B reacts 550 N. What signed applied couple is consistent with the departure from simple force sharing?

▼ **Subtract the force-only support moment**

A force-only prediction for B is

$$780(60/150) = 312\,\mathrm{N}.$$

The excess 238 N corresponds to a couple of magnitude 35.7 N m in the direction increasing B. With a 50 mm pitch radius that magnitude would imply 714 N axial force if off-axis thrust were its only source. Check the thrust bearing directly. Misaligned couplings or another overhung force can give the same two transverse reactions, so the inversion identifies a net moment rather than its unique origin.

Tutorial · Explain a life sensitivity

For an ideal ball bearing the reference rating model gives life in millions of revolutions as $(C/P)^3$. A catalogue combined-load calculation increases effective load from 1.00 to 1.20 kN at unchanged rating. Estimate the life ratio and explain what data the ratio omits.

▼ **Keep the bearing rating and the statistical meaning fixed**

The ratio is $(1/1.20)^3 = 0.579$, a reduction of about 42 percent in the rating life. This is a population-based rating at a chosen reliability under specified conditions. Lubrication, contamination, preload, fit, load cycles and installation can change service results. Comparing against a test measured in hours also requires shaft speed. If speed doubles at the same effective load, the reference life in revolutions stays the same while the corresponding time halves.

MIT Marine Power and Propulsion · helical gear lecture

Design and analysis of machine components · 3. Sliding wear, real contact area and life inference from a coupon

A sliding carriage develops measurable clearance while its normal force remains almost constant. Derive a practical volume-loss rule from microscopic contacts, estimate its coefficient from a mass trace, and test whether a single fitted value can be transferred to another load history. Geometry, material condition and instrument resolution enter at different stages.

Count interaction volume per travelled distance

The apparent contact area of two nominally flat surfaces is usually much larger than the sum of microscopic load-bearing patches. For an elementary model suppose these patches deform plastically at mean pressure close to a hardness H of the softer body. Their summed real area is then approximately W/H at applied normal force W. As the surfaces slide by distance ds they sweep an interaction volume proportional to this area times distance. Only a fraction of microscopic encounters leads to detached material. Absorb this probability and unknown geometry into the dimensionless empirical coefficient k. Integrating the resulting increment gives the Archard form:

$$dV = k\frac{W}{H}\,ds,$$

$$V = \frac{kWs}{H},$$

$$h_{\mathrm{mean}} = \frac{V}{A_{\mathrm{nominal}}}.$$

Force divided by pressure has units of area. The interaction-volume picture explains the dimensional structure without predicting the coefficient for a new material pair. Some tables give a dimensional specific wear rate $V/(Ws)$; it already includes the inverse hardness. Its tabulated units may be cubic millimetres per newton metre. Multiplying that number by another inverse hardness counts hardness twice. State which body loses the quoted volume. In a pair of dissimilar materials the track and the slider may have quite different loss rates.

A teaching coupon carries 100 N over 1 km at hardness 2 GPa and fitted coefficient 2×10^{-5}. The model gives 10^{-9} cubic metres, exactly one cubic millimetre. Uniform removal over 50 square millimetres would lower the surface by 20 micrometres. A profilometer on a narrow groove could report a much deeper maximum while its integrated volume still agreed. Report both track shape and volume when the function of a part depends on clearance at one point.

Separate distance, time and the running-in transition

For changing conditions the differential statement is more useful:

$$V(t) = \int_0^t \frac{k(t')W(t')}{H(t')}\,\lvert v_{\mathrm{rel}}(t')\rvert\,dt'.$$

Motion reversals contribute positive distances. A reciprocating slider returning to the starting position has zero net displacement and can have substantial wear. If force varies with speed, the average of their product is generally different from the product of their averages. A loss model calibrated during steady sliding may fail near each reversal where a lubricating film collapses. Integrating only the relative movement at the actual interface excludes contact portions undergoing nearly pure rolling.

Suppose a balance resolves 0.5 mg and the coupon density is 7.8 mg per cubic millimetre. A loss of one cubic millimetre gives 7.8 mg signal. The single-reading resolution is equivalent to about 0.064 cubic millimetres. A before/after difference contains uncertainties from both measurements. Cleaning, oil retention, corrosion and material transferred from the mating part can change mass independently of track removal. Repeated weighing after identical cleaning and a stationary witness coupon distinguish some of those effects. A single extremely precise reading cannot correct a consistent cleaning bias.

Vary follower pressure angle and examine why guide normal load changes over a stroke before integrating wear.

Examine a case in which the mean-load approximation becomes exact

In the plot the velocity increases after thirty minutes, while both half-hour loads can be set separately. Observe the change of slope at the switch. The slope measures instantaneous volume loss per time, so its jump follows the product of load and speed. With the default pair of loads the speed increase just compensates for the smaller load, leaving the integrated history straight despite the change of regime assumed by the operator. The comparison based on separate averages still runs faster.

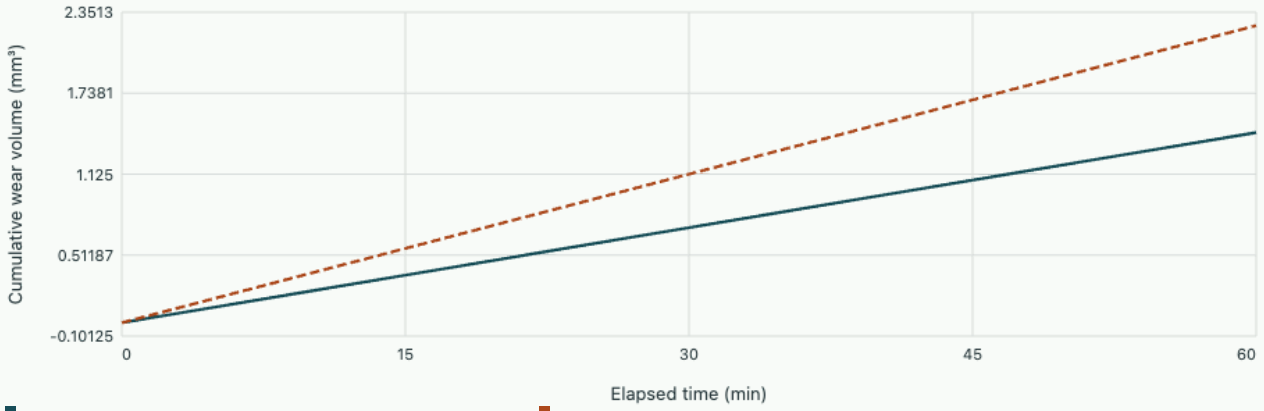
Open the live experiment

Try the calculation

First interval load (N) **200**

Second interval load (N) **50**

Hardness (GPa) **2**



Two successive operating conditions Separate average load and average speed

After one hour the integrated history predicts 1.440 mm³. Multiplying separate averages predicts 2.250 mm³ (56.3% difference).

Set the two loads equal. The accumulated trace now has two slopes, yet its final value coincides with the separate-average prediction because force is constant and integrating the velocity simply counts total distance. This counterexample helps separate agreement at one endpoint from agreement during the programme. An inspection at the changeover could discriminate two histories which yield the same final mass loss.

Tutorial · Calibrate the dimensionless coefficient

A pin loses 3.90 mg after 500 m under 100 N. Its density is 7.80 mg per cubic millimetre and hardness is 2 GPa. Infer the coefficient and predict the volume after another 1.5 km if the wear regime persists.

▼ **Convert mass into a volume before fitting**

The first loss is half a cubic millimetre, or 5.0×10^{-10} cubic metres. Solving $k = VH/(Ws)$ gives 2.0×10^{-5}. Three times the sliding distance then removes another 1.5 cubic millimetres at unchanged conditions. A straight line through multiple later measurements supports constant rate; the intercept can reflect substantial running-in wear before that linear stage. Do not force the post-running-in line through the origin unless observations justify that endpoint.

Tutorial · Compare service programmes with identical mean load and speed

Over equal half-hour intervals a guide first carries 200 N at 0.2 m/s then 50 N at 0.8 m/s. Another test uses average force and average speed throughout the same hour. Compare their modelled losses.

▼ **Integrate the load–distance product**

Each half of the first programme has force times speed equal to 40 N m/s, so its integrated exposure is 144,000 N m. The second has $125 \times 0.5 = 62.5$ N m/s throughout and reaches 225,000 N m. The second predicted volume is 1.5625 times the first if hardness and coefficient stay fixed. The first programme anticorrelates force and speed, making a product of separately averaged values inappropriate. Differences in contact temperature could change the coefficient and must be measured separately before comparing real coupons.

Tutorial · Diagnose an apparent change of mechanism

A test at fixed force and speed shows cumulative loss 0.9, 1.1 and 1.3 cubic millimetres after 100, 200 and 300 m. A colleague fits the end-to-end origin slope. Which prediction would you test at 400 m and why?

▼ **Compare local slopes and collect an independent reading**

The final two intervals both lose 0.2 cubic millimetres per 100 m. Continuing that local trend predicts 1.5 cubic millimetres at 400 m. The origin-to-300 m slope predicts about 1.73. The difference is experimentally distinguishable only with adequate repeatability. Larger initial roughness, a forming transfer layer, drift in cleaning or a change of counterface can each generate an offset. Inspect surface images and repeat new and preconditioned coupons. A single cumulative curve suggests a running-in stage but does not prove its microscopic mechanism.

École polytechnique fédérale de Lausanne · A parameter-free mechanistic model of the adhesive wear process of rough surfaces in sliding contact

Design and analysis of machine components · 4. Fatigue crack growth, optical inspection and the meaning of remaining life

A polished test coupon contains a measurable crack and experiences repeated tensile cycles. A stress–life curve for a polished, initially uncracked specimen cannot directly give the remaining growth interval from this measured state. Use calibrated crack-growth data to predict a laboratory observation and quantify what the inference depends on.

Connect load history to a crack-tip driving range

Outside a small process zone the leading elastic opening stress near an ideal crack tip scales like $K_I/\sqrt{2\pi r}$. The dimension of K_I is stress times square root of length. In a geometrically similar family under remote uniaxial tension, linearity in stress and similarity in dimensions give

$$K_I = Y\sigma\sqrt{\pi a},$$

where the dimensionless geometry factor depends on crack shape, relative crack length and loading. Define whether a denotes crack depth, half of a central crack, or an edge crack length before using a calibration.

Under constant-amplitude cycles at a fixed stress ratio, stable intermediate-rate measurements often obey the empirical Paris relation. With approximately constant Y, separation of variables explains why successive millimetres of growth take different numbers of cycles:

$$\frac{da}{dN} = C(\Delta K)^m = C(Y\Delta\sigma\sqrt{\pi})^m a^{m/2},$$

$$N = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(Y\Delta\sigma\sqrt{\pi})^m(1-m/2)}.$$

For exponent two the integral instead gives a logarithm. The units of the coefficient depend on both exponent and stress-intensity units. A coefficient expressed with MPa and metres cannot be inserted unchanged with Pa and millimetres. Use positive growth range and verify a physically increasing final crack length gives positive cycles. Near threshold, near fast fracture, at changing load ratio or in large-scale yielding the assumptions leading to this simple expression require reconsideration.

Predict two distinctly different stages of growth

Take an illustrative constant geometry factor 1.10, range 80 MPa, coefficient 10^{-11} in metres per cycle per cubed MPa square-root metre, and exponent three. The driving range grows from 4.93 to 9.87 MPa square-root metres as the crack grows from 1 to 4 mm. Integration gives 833,350 cycles. The instantaneous growth rate at 4 mm is eight times the rate at 1 mm. Replacing the whole history by the initial rate would significantly overpredict the number of cycles. Using the final rate would err in the opposite direction.

At maximum tensile stress 120 MPa, the maximum stress intensity at 4 mm is 14.8 MPa square-root metres. A terminal criterion based on a known fracture toughness uses the maximum intensity while a growth calculation uses the range. Those quantities answer different experimental questions. Finite width can raise the geometry factor with increasing crack size. In that case integrate $1/[C\Delta K(a)^m]$ on the measured geometry instead of treating the earlier constant value as universal. Refine the length increments until the predicted cycle count is stable compared with optical measurement uncertainty.

Link an optical length to a calibrated loading history

Use the same reference marks and magnification at each inspection and record loading cycles including interruptions. Surface observations describe a surface trace, which can grow differently from a subsurface front. Locally estimate growth rate from two nearby length readings divided by their cycle separation and assign a representative crack length within the interval. Widely spaced readings average a rapidly changing rate; too narrow a spacing lets registration error dominate the difference. Repeat the fit with both larger and smaller intervals and inspect residuals.

Compare the bonded-overlap stress study with a nominal uniform-stress assumption. Its smooth transfer field supplies loading information upstream of a crack-tip analysis.

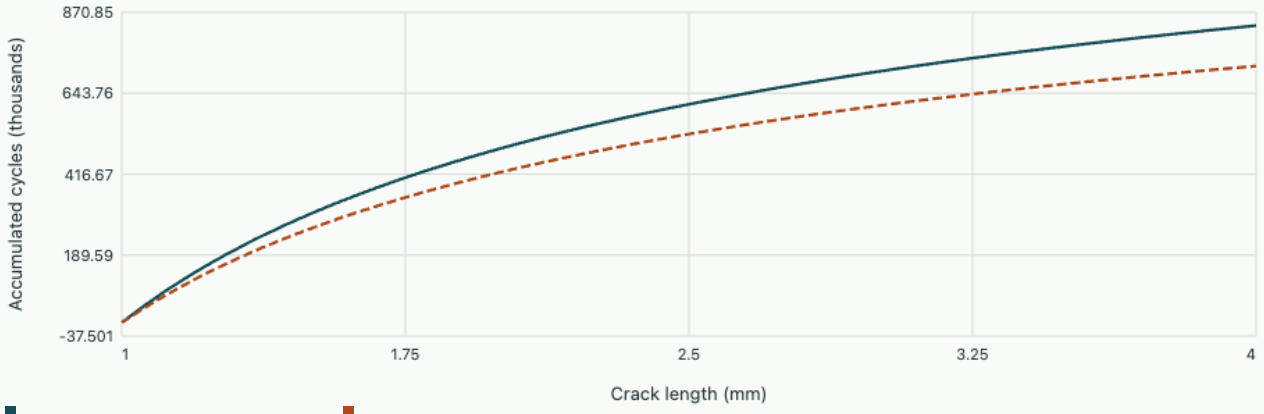
Use the inverse crack-length plot to plan a coupon observation

Plot accumulated cycles vertically and crack length horizontally. The slope gives the number of additional cycles per increment of length, which falls as the crack grows. A flatter curve near the end therefore means faster crack growth. Increasing stress range compresses the entire vertical axis because the constant-amplitude Paris equation is separable. Changing only the initial crack size removes an especially slow portion of the growth history. Try a small change at the initial end and then the same absolute change at the final end; compare how strongly each affects predicted cycles.

Open the live experiment

Try the calculation

Stress range (MPa)	80
Initial crack length (mm)	1
Final crack length (mm)	4



Growth over the selected length interval takes 833,349 cycles at the fitted intermediate-growth law. The endpoint growth rate increases by a factor 8.00.

On a real finite-width coupon the geometry factor can also change with length. That change alters the curvature of the graph and cannot generally be represented by rescaling the cycle axis. A lack of collapse after rescaling curves from two different loads may indicate a changing stress ratio, crack closure or a load-dependent material response. Recheck cycle counting and optical length units before selecting a more elaborate law.

Tutorial · Partition the predicted interval in half by crack length

For the coupon above estimate cycles spent between 1 and 2 mm and between 2 and 4 mm. Explain why the longer second length interval takes less time.

▼ **Compare the differences of reciprocal square roots**

For exponent three the cycle count is proportional to $a_i^{-1/2} - a_f^{-1/2}$. The first interval occupies about 58.58 percent of the total, or 488,200 cycles. The second takes about 345,200 cycles. As the stress-intensity range rises with square root of crack size, the cubed growth law accelerates faster than length itself. These fractions are independent of the common coefficient provided it truly stays the same. A rate plot offers a separate check: doubling length at constant stress increases the local rate by $2^{3/2}$.

Tutorial · Determine sensitivity to one calibration error

Load calibration overestimates the true stress range by five percent. All other quantities are independently correct. Find the resulting error in the predicted growth interval, and compare with a ten-percent coefficient error.

▼ **Use the inverse growth-law dependence**

Using the high stress gives a cycle count multiplied by $1.05^{-3} = 0.864$, about 13.6 percent low. Using a coefficient ten percent high gives a factor of $1/1.10 = 0.909$, about 9.1 percent low. A stress calibration error can also bias the fitted coefficient if the same load cell supplied calibration data. Shared errors should then be propagated jointly rather than assumed independent. Agreement obtained using one calibration frame does not guarantee transfer to a second one.

Tutorial · Distinguish no detected crack from a zero-length crack

An instrument detects a reference flaw of 0.8 mm only intermittently. A student inserts zero as the initial size after an image returns no indication and reports an infinite life for exponent three. Explain the mathematical result and design a more informative coupon test.

▼ **Identify the extrapolation and its observational limit**

The integral diverges at zero because the ideal long-crack power law predicts an arbitrarily small rate at vanishing crack length. The experiment supplied only a non-detection at finite resolution. Crack initiation, small-crack behaviour and detection probability lie outside the fit. Seed coupons with measured flaws spanning the optical threshold, take repeated coded images, and compare against a second length measurement. Report a detection curve and a conditional range of growth times for resolved starting sizes. That produces a testable inference without interpreting an undetected flaw as zero.

MIT 3.35 · fatigue crack growth lecture and laboratory resources

Design and analysis of machine components · 5. How does a converging oil film support a load?

Two plates have a small fixed angle. The lower plate moves while the upper pad stands still. Derive the pressure from volume conservation and local shear, then change film height and speed to see why a wedge develops lift.

Combine pressure flow and wall-driven flow before imposing continuity

Let x point along the moving surface from the wide inlet h_1 to the thin outlet h_2. The film has constant density and viscosity μ. It is shallow compared with its length L, and the pad is broad enough to neglect side leakage. Local steady momentum balance is

$$\mu u_{yy} = dp/dx.$$

Integrate across the gap with $u(0) = U$ and $u(h) = 0$. The linear boundary-driven velocity combines with a parabolic pressure-driven term. Integration of that velocity gives the volume flow per unit width

$$q = Uh/2 - \frac{h^3}{12\mu}\frac{dp}{dx}.$$

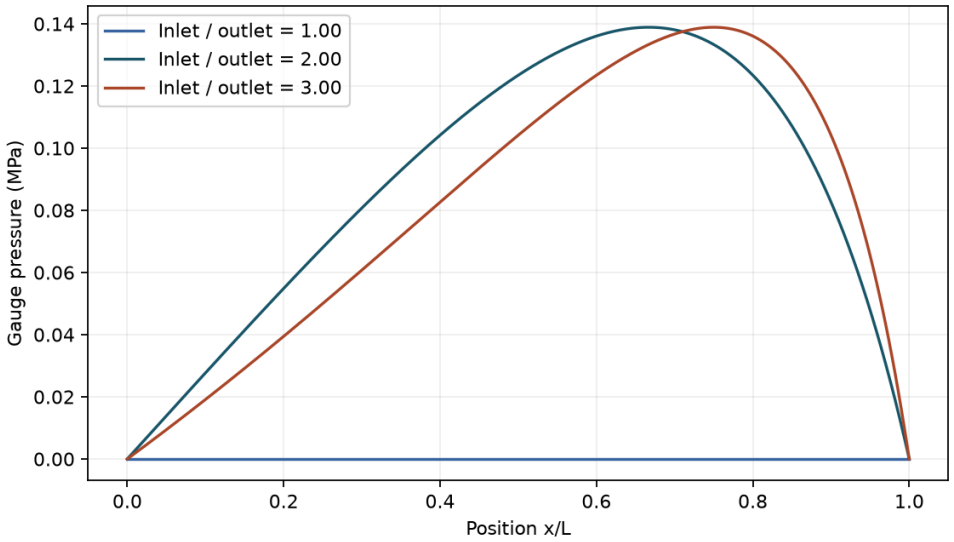
The same q passes every station. Introduce $h_* = 2q/U$, making $dp/dx = 6\mu U(h - h_*)/h^3$. Equal ambient gauge pressures at both ends require the integral of the gradient over x to vanish. For a linearly tapering film convert integration from x to h and obtain $h_* = 2h_1h_2/(h_1 + h_2)$. The pressure maximum lies where $h = h_*$. Integrating once more gives a direct pressure profile:

$$p = \frac{6\mu UL}{h_1 - h_2}\left[\frac{1}{h} - \frac{1}{h_1} - \frac{h_*}{2}\left(\frac{1}{h^2} - \frac{1}{h_1^2}\right)\right].$$

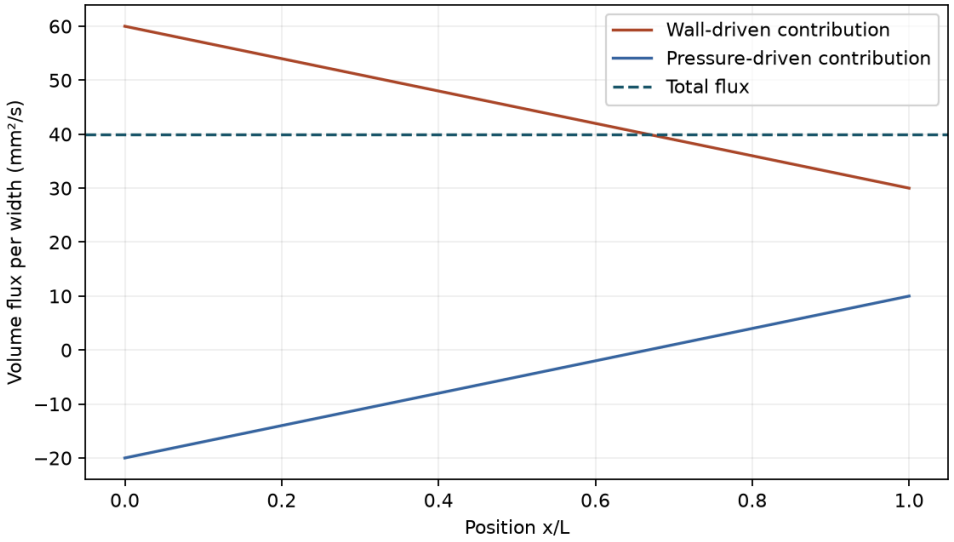
This form subtracts nearby terms in nearly parallel gaps. The notebook uses the equivalent factored result $p = 6\mu UL(h_1 - h_2)s(1 - s)/[(h_1 + h_2)h^2]$, where $s = x/L$. It remains well behaved as the slope tends to zero.

Measure the pressure gradient where the Couette flux matches the actual flow

Choose inlet and outlet heights of 120 and 60 µm, length 40 mm, sliding speed 1 m/s and viscosity 0.050 Pa s. Then $h_* = 80$ µm and the flux per unit width is 4.00×10^{-5} m²/s. The pressure maximum is two thirds of the length from the inlet. For these data it is approximately 0.139 MPa. At that point the pressure-driven part of the flow vanishes locally even though the accumulated pressure above ambient is greatest. This is the distinction between a local gradient and an absolute load-carrying pressure.



The outlet thickness is held fixed in this comparison. A parallel gap carries fluid by shear while producing zero pressure load under the stated boundary conditions.



The pressure-driven contribution opposes motion near the wide inlet and assists it nearer the outlet. Its sign changes at the pressure maximum.

Exercise · scale both clearances

Double inlet and outlet thickness while holding taper ratio, viscosity, length and speed fixed. What becomes of the pressure and the flow?

▼ Work through the solution

The factor $(h_1 - h_2)/(h_1 + h_2)$ is constant, while the squared local height quadruples. Pressure and load per width fall to one quarter. The harmonic-mean height h_* doubles, so the through-flow doubles.

Exercise · interpret a reversed pad

What does the algebra predict for a diverging gap with the same wall direction? Under what condition does that extrapolation fail?

▼ Work through the solution

The sign of $h_1 - h_2$ reverses, giving negative gauge pressure between the equal-pressure boundaries. Small subambient pressures can exist in a filled liquid. Once the absolute pressure reaches the cavitation threshold or gas enters the film, a full-film solution cannot represent the cavity or its mass balance. The notebook therefore keeps its default sweep in the converging regime.

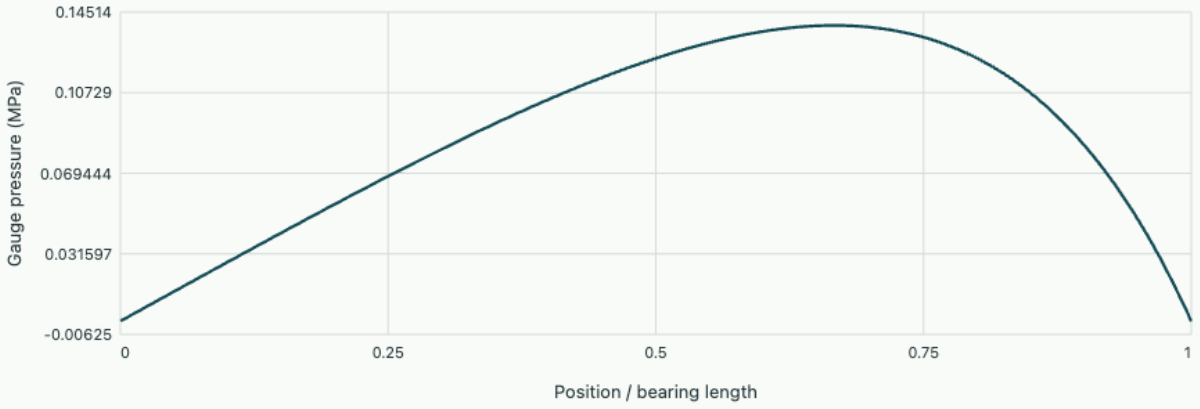
Interactive calculation · Slider pressure

Change the physical and numerical inputs of the notebook model. Compare the updated curves and balance checks with the recorded notebook figures. Each plot uses the same equations and assumptions documented in that notebook; its readout names any analytical or reduced-model reference.

Open this live calculation

Try the calculation

Inlet / outlet gap	2
Outlet gap (µm)	60
Sliding speed (m/s)	1



Reynolds pressure

Peak pressure 0.1389 MPa at x/L=0.667. Flux 40.000 mm²/s. Length40 mm, viscosity0.05 Pa s, incompressible isoviscous film, ambient-pressure ends.

Download this example’s editable notebook

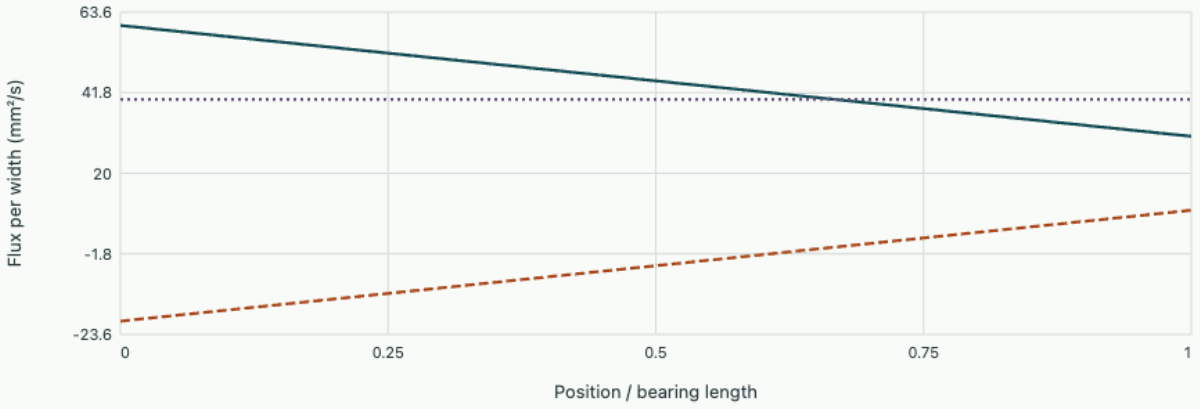
Interactive calculation · Slider flux

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Download this example's editable notebook

Open editable notebook Download .ipynb

Design and analysis of machine components · 6. From a laboratory fatigue curve to a machined shaft

A fatigue limit measured on a polished specimen does not directly describe a shoulder on a production shaft. Track where each correction enters and how the measured torque or bending history changes the inferred life.

Keep coupon, component and nominal stress distinct

Take a quoted fully reversed specimen strength $S'_f(N_*)$ at a stated reference life N_*. A component with a different finish, stressed volume, environment or survival requirement has different statistics of crack initiation. Here use experimentally supplied finish and size factors k_a, k_b for this process and diameter. They act on allowable amplitude. Factors calibrated at one life need supporting data before use over another regime.

$$S_f(N_*) = k_a k_b S'_f(N_*),$$

$$K_f = 1 + q(K_t - 1).$$

The elastic concentration K_t is the local-to-nominal peak from the geometry. Notch sensitivity q transfers the nominal alternating component to an effective fatigue amplitude. Its values zero and one recover no notch penalty and the full elastic concentration. Keep surface correction on the material side and stress concentration on the demand side. Applying the same notch debit to both counts it twice.

Move a load cycle to the reference mean

For a cycle with nominal maximum and minimum stresses, their half-difference is σ_a and their mean is σ_m. With elastic local response, a specified tensile-mean Goodman approximation gives a fully reversed equivalent demand. Adopt zero benefit from compressive mean for this example. A separate maximum-stress check against yield must precede this calculation.

$$a_{eq} = \frac{K_f \sigma_a}{1 - K_t \max(\sigma_m, 0)/S_u},$$

$$N_f = N_* \left(\frac{S_f(N_*)}{a_{eq}} \right)^b.$$

Take $S_u = 650$ MPa, $S'_f = 280$ MPa at a million cycles, $k_a = 0.8$, $k_b = 0.88$, $q = 0.8$, $K_t = 1.65$ and exponent $b = 5$. A 100 MPa amplitude and 30 MPa mean give $K_f = 1.52$, component strength 197.1 MPa, and equivalent demand 164.5 MPa. The illustrative power law predicts about 2.47 million repetitions. It has no endurance knee. Extrapolation to small stresses would require additional material data.

What did the heat treatment change?

A surface-hardened shoulder can have hardness and residual stress gradients across the crack-initiation zone. A coupon from the core measures neither directly. Select data from the production condition, measure the local radius and identify any residual tensile mean before assigning corrections from a handbook.

Turn a measured record into cumulative damage

Rainflow counting pairs stress reversals to recover amplitudes, means and fractional counts. Analyze an explicitly finite record and retain its half cycles. A periodic mission should instead be closed consistently across its boundary. Linear accumulation then sets $D = \sum_i n_i / N_i$ for this model. Sequence sensitivity and residual stress relaxation require independent checks.

Interactive experiment · Move a load cycle across the Goodman diagram

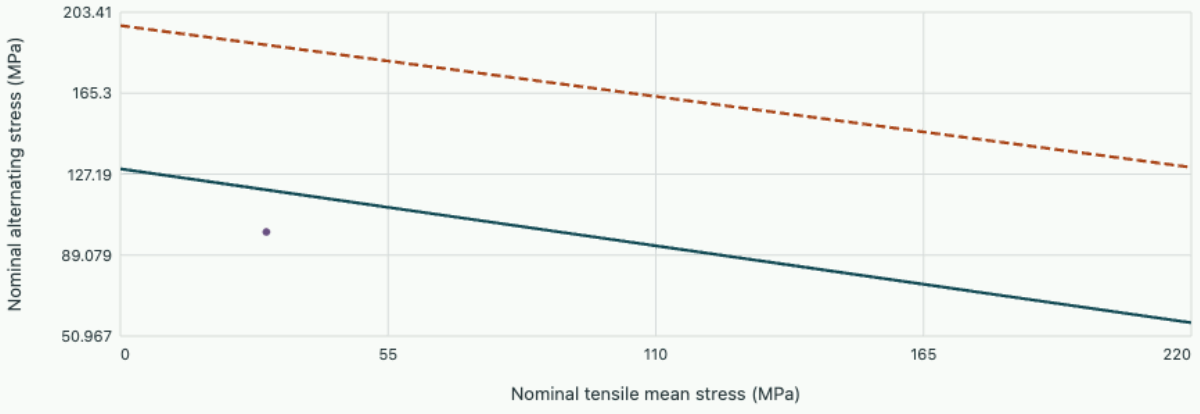
Set the mean and alternating nominal stress to place a single constant-amplitude cycle in the diagram. Its boundary corresponds to one million cycles in the illustrative material law above. Change the surface factor and elastic concentration separately to see which acts on the available strength and which on the applied demand. The unnotched comparison uses the same finish and size corrections.

Begin with a 30 MPa mean and a 100 MPa amplitude. Raising the surface factor from 0.80 to 0.90 multiplies predicted life by approximately 1.80. Increasing the mean stress lowers the admissible amplitude even when the stress range is unchanged. This interactive diagnostic treats one repeating cycle. The OneSimulation notebook below counts the distinct cycles of a finite load record before accumulating damage. The displayed law assumes an elastic local response, includes no endurance knee and needs a separate yield check.

Open this experiment online

Try the calculation

Surface factor	**0.8**
Elastic concentration Kt	**1.65**
Nominal mean stress (MPa)	**30**
Nominal alternating stress (MPa)	**100**



One-million-cycle Goodman boundary
Same component without a notch
Selected nominal cycle

Kf = 1.520, corrected strength 197.12 MPa, equivalent reversed amplitude 164.53 MPa. Illustrative constant-amplitude life: 2.469e+6 cycles; damage per cycle 4.051e-7. This finite-life model has no endurance knee and does not check yielding.

Continue in the OneSimulation notebook · Download its Python model

OneSimulation experiment · Changing the measured finish factor changes damage from the same counted load record

Run the notebook with OneSimulation 0.2.0, NumPy, SciPy and Matplotlib. Change the input data and compare your run with the recorded result.

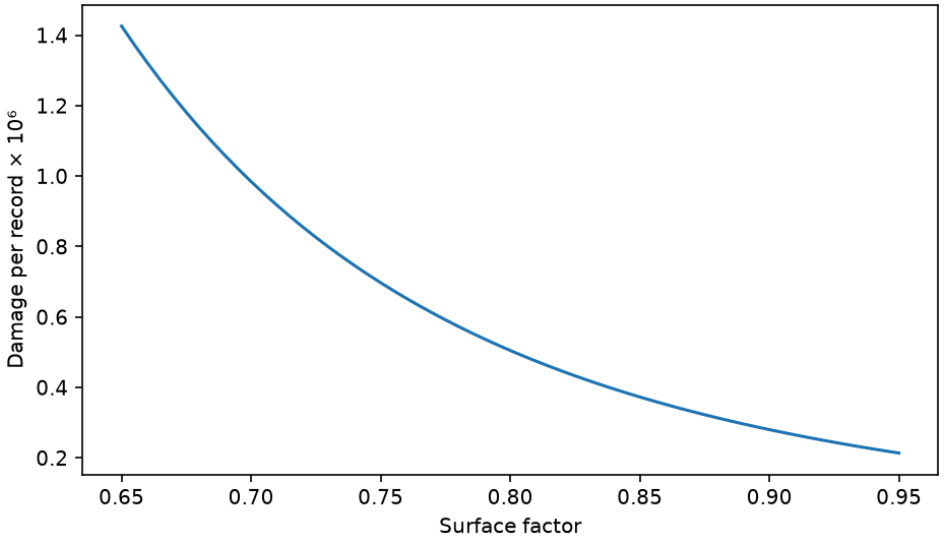
```
"""Compare the effect of measured surface factors at one shoulder under a finite record."""
import numpy as np
import matplotlib.pyplot as plt
from onesim.physics.solid_mechanics.fatigue.models.cycles.cycle_counting import fatigue_cycles

nominal = np.array([0,110,-40,85,-65,130,-55,80,0.0])  # MPa
cycles = list(fatigue_cycles(stress=nominal))
surface = np.linspace(0.65, 0.95, 61)
kt, notch_sensitivity, ultimate = 1.65, 0.8, 650.0  # ultimate in MPa
kf = 1+notch_sensitivity*(kt-1)
# Illustrative calibrated finite-life power law. Reference life: one million cycles.
def block_damage(ka):
    strength = 280*ka*0.88  # MPa at reference life, after size correction
    damage = 0.0
    for cycle in cycles:
        mean = kt*max(cycle.mean_stress, 0)
        alternating = kf*cycle.stress_amplitude/(1-mean/ultimate)
        damage += cycle.count/1e6*(alternating/strength)**5
    return damage

damage = np.array([block_damage(value) for value in surface])
print("counted cycle equivalents", sum(cycle.count for cycle in cycles))
for ka in [0.70, 0.90]:
    print(f"surface factor {ka:.2f}: damage per record = {block_damage(ka):.4e}")
```

Reference run

```
counted cycle equivalents 4.0
surface factor 0.70: damage per record = 9.8463e-07
surface factor 0.90: damage per record = 2.8025e-07
```



Changing the measured finish factor changes damage from the same counted load record

> Download editable notebook · Download Python script

Exercises on fatigue evidence

Polish the previous shaft so that k_a increases from 0.80 to 0.90. How does the predicted life change at the same cycle and exponent?

▼ **Worked solution**

At fixed mean correction and amplitude, N_f scales as k_a^5. The multiplier is $(0.90/0.80)^5 = 1.80$. Changing K_t as well would require a separate geometry update.

Two records have equal peaks at 160 MPa. The first cycles between 0 and 160, the second between –160 and 160 MPa. Which has larger corrected demand here?

▼ **Worked solution**

The first gives $1.52(80)/(1 - 1.65(80)/650) = 152.6$ MPa. The second gives $1.52(160) = 243.2$ MPa. A peak-only count would discard the dominant difference in range.

Further reading · Budynas and Nisbett, Shigley's Mechanical Engineering Design, 10th ed., Chapters 2, 5 and 6.

Design and analysis of machine components · 7. Will tooth-root bending or flank contact govern a gear pair?

The same torque loads the tooth root and the mating flanks. Calculate two distinct demands, carry the same duty blocks through both, and see why doubling the face width gives different gains.

Resolve a torque into tooth force

Take straight external spur gears with a pitch diameter d and pressure angle α. At the pitch circle, transmission of torque T fixes $F_t = 2T/d$. The nominal normal force is $F_n = F_t/\cos\alpha$. Use one conservative single-tooth load factor K for the example. Real cyclic mesh sharing, alignment, speed and surface finish require data specific to the teeth.

For root bending, replace the tooth by a cantilever section. Its bending stress scales as $6F_t\ell/(bt^2)$ for lever arm ℓ, face width b and root thickness t. A dimensionless shape factor Y collects the section geometry into a module-based estimate. For contact take elastic nonconforming line contact with reduced modulus E' and effective local radius R'. Integrating the Hertz pressure $p_0\sqrt{1-(x/a)^2}$ over width $2a$ gives $KF_n/b = \pi a p_0/2$. Compatibility with the local quadratic gap gives $a^2 = 4KF_nR'/(\pi bE')$. Eliminating a gives

$$\sigma_F = \frac{KF_t}{bmY},$$

$$p_H = \sqrt{\frac{E'KF_n}{\pi bR'}}.$$

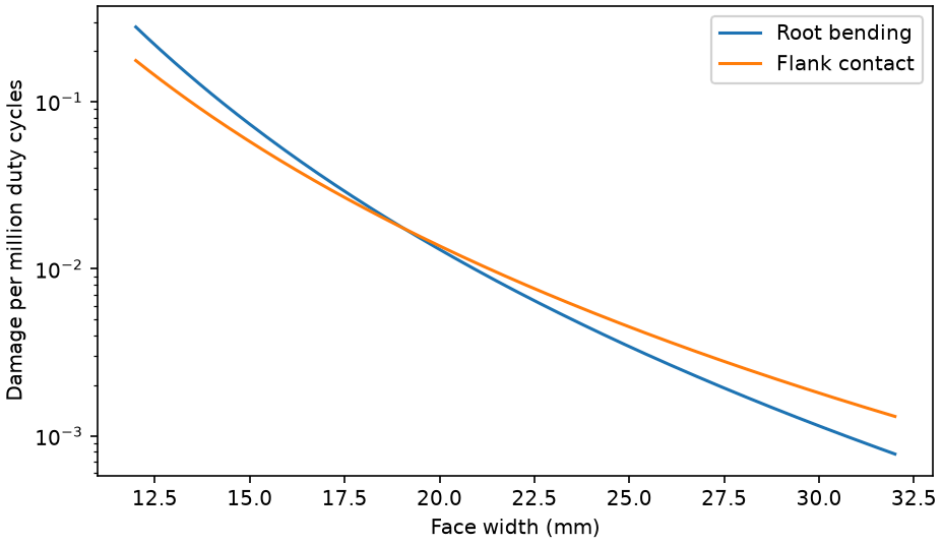
Follow one reducer through two life checks

Use $T = 80$ N m, $d = 80$ mm, $m = 2.5$ mm, $b = 16$ mm, $Y = 0.32$, $\alpha = 20^\circ$, $K = 1.30$, $E' = 115$ GPa, and $R' = 8$ mm. The nominal tangential force is 2000 N. The model gives $\sigma_F = 203.1$ MPa and $p_H = 889.5$ MPa. The contact pressure is a compressive surface quantity and its pitting limit must come from a contact-fatigue test.

For an illustrative material pair take $N_F = 10^7(220\,\mathrm{MPa}/\sigma_F)^6$ and $N_H = 10^7(950\,\mathrm{MPa}/p_H)^{10}$. Spend 80% of cycles at full torque and 20% at half. Since root stress scales with T and contact pressure with $\sqrt{T}$, damage per million cycles is $0.1(203.1/220)^6[0.8 + 0.2/64] \simeq 0.050$ for bending and $0.1(889.5/950)^{10}[0.8 + 0.2/32] \simeq 0.042$ for contact. In this assumed pair, root fatigue uses more of its life. A change of flank hardness can change which limit controls.

Where an industrial rating requires additional evidence

These two models explain scaling and identify required inputs. A standard rating requires the applicable tooth geometry, quality, dynamic and distribution factors, material condition, lubrication, life and reliability provisions. The illustrative constants are not a substitute for those inputs.



Root and flank damage have different sensitivities to face width. The worked 16 mm gear is root-governed with the example material data.

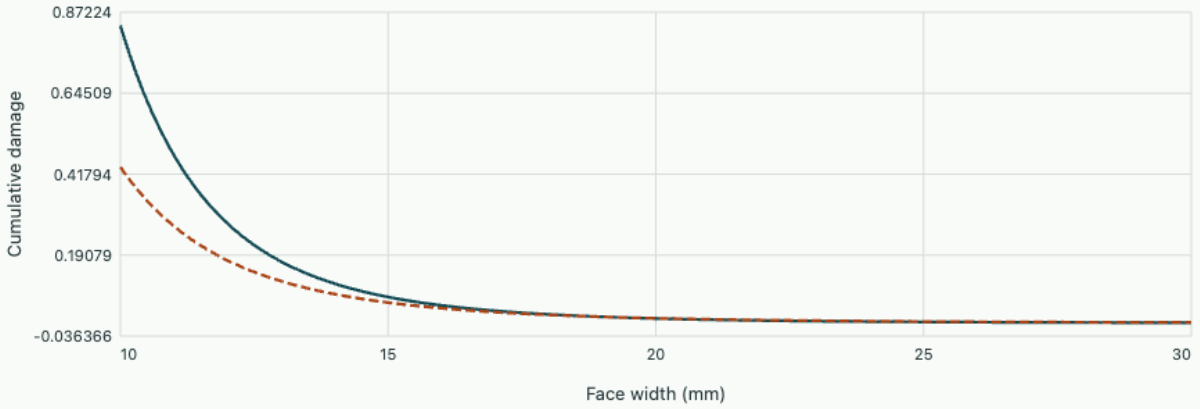
Interactive calculation · Carry a duty cycle through root and flank laws

Change torque and mission length, then compare the different face-width exponents. Half-torque cycles fill the remainder of the specified full-torque fraction.

Open this live calculation

Try the calculation

Full torque (N m)	80
Full-torque cycle fraction	0.8
Mission (million cycles)	1



At16 mm, root stress 203.125 MPa and contact pressure 889.534 MPa. Half-torque remainder; the example fatigue laws have exponents6 and10.

Size the face width for one duty and two failure mechanisms

Retain the pitch diameter, module, normal force, pressure angle and radii of the worked gear. A design brief specifies 20 million pinion load cycles, of which 80% are at 80 N m and 20% at 40 N m. It supplies an application load multiplier $K_A = 1.15$, a measured dynamic amplification $K_v = 1.10$, and face-distribution factors $K_{bF} = 1.18$ for root stress and $K_{bH} = 1.12$ for pressure. These illustrative factors replace the earlier single K and apply to load. Define $K_F = K_A K_v K_{bF} = 1.4927$ and $K_H = K_A K_v K_{bH} = 1.4168$. Root stress is proportional to K_F. Contact pressure is proportional to $\sqrt{K_H}$.

On the material side, take 10-million-cycle reference stresses of 260 MPa at the root and 1050 MPa in contact. The design data supply two independent strength multipliers for each mechanism. Root strength is $S_F = 260(0.95)(0.97) = 239.59$ MPa, and flank strength is

$S_H = 1050(0.98)(0.95) = 977.55$ MPa. Use the laboratory exponents 6 and 10 against these corrected strengths. This completely defines the teaching model from measured loads to allowable cumulative damage $D_{lim} = 0.60$. The factors are design inputs, with no claim to implement a particular industrial standard.

At $b_0 = 16$ mm the full-load root stress is 233.23 MPa and the full-load peak contact pressure is 928.64 MPa. The half-torque root stress is 116.62 MPa; the half-torque pressure is 656.64 MPa. Summing counted cycles divided by their mechanism-specific lives gives

$$D_F(b_0) = 2(0.8 + 0.2/2^6)(233.23/239.59)^6 = 1.367,$$

$$D_H(b_0) = 2(0.8 + 0.2/2^5)(928.64/977.55)^{10} = 0.965.$$

Both exceed the design limit despite the full-load stresses being below their reference strengths. At fixed distribution factors $D_F \propto b^{-6}$ and $D_H \propto b^{-5}$. The separate required widths are $16(1.367/0.60)^{1/6} = 18.35$ mm and $16(0.965/0.60)^{1/5} = 17.60$ mm. Choose a 20 mm available width. Its damages are 0.358 and 0.316. The margin is then traceable to one specified mission, one set of material data and one geometry. Check each gear separately where tooth counts and hence cycle counts differ.

A contact pattern test at 20 mm finds that the flank-distribution factor is 15% greater than assumed. Does the adopted width still meet the stated target? What width would restore it if all other inputs stayed fixed?

▼ **Worked solution**

Pressure grows by $\sqrt{1.15}$. Damage therefore grows by 1.15^5, taking the flank result from 0.316 to 0.636, just above 0.60. Required width becomes about $20(0.636/0.60)^{1/5} = 20.23$ mm. The original root result is unchanged under the problem’s assumption. A new width needs a new contact-pattern check if alignment changes.

Exercises on design changes

Double the face width while keeping load sharing uniform. What happens to each computed damage?

▼ **Worked solution**

Root stress halves, reducing bending damage by $2^6 = 64$. Contact stress falls by $\sqrt{2}$, reducing contact damage by $2^5 = 32$. A widened but poorly aligned mesh can lose much of this benefit.

Why might a gear with safe tooth bending still exhibit pitting after its lubricant changes?

▼ **Worked solution**

The root check depends chiefly on tangential load and root section. Lubrication, film thickness, roughness and local sliding influence the durability of the contacting surfaces. A bending margin does not supply a contact life or a wear prediction.

Further reading · Budynas and Nisbett, Shigley’s Mechanical Engineering Design, 10th ed., Chapters 13–14 and 18.

Design and analysis of machine components · 8. Redirecting a shaft line with bevel gears and worm drives

Two transmissions can deliver the same speed reduction around a right angle yet require very different bearings and cooling. Derive the directions of the loads and the dependence of efficiency on lead angle.

Use the common pitch-cone apex

For perpendicular intersecting shafts and straight bevel teeth, matching peripheral speeds at a common cone distance R implies $d_1/d_2 = z_1/z_2$. The two cones meet along a shared generator, so $\delta_1 + \delta_2 = 90^\circ$ and $\tan\delta_1 = z_1/z_2$. Use the mean pitch diameter at the centre of the face to calculate forces. The transverse pressure-angle component $F_t \tan\alpha$ acts normal to the pitch cone. Resolve that component parallel and perpendicular to the shaft:

$$F_t = \frac{2T}{d_m},$$

$$|F_a| = F_t \tan\alpha \sin\delta,$$

$$|F_r| = F_t \tan\alpha \cos\delta.$$

For a 20-tooth pinion driving a 40-tooth wheel at 90°, $\delta_1 = 26.57^\circ$. At mean pinion diameter 64 mm and torque 80 N m, $F_t = 2500$ N. With a 20° pressure angle this gives radial and axial magnitudes of 814 N and 407 N. The wheel sees the equal and opposite contact force in space. Its own shaft axes exchange radial and axial roles. Use the direction of rotation and the identified tooth flank to assign signs.

Unroll one turn of the screw

A cylindrical worm with z_s starts advances the wheel z_s teeth each revolution. Consequently $i = z_w/z_s$. Unwrap the mean worm cylinder into a plane. One turn advances axial lead L over distance πd_w, so $\tan\gamma = L/(\pi d_w)$. For sliding contact represented by an effective friction angle $\phi = \arctan\mu_{eff}$, equilibrium along the inclined plane requires input effort proportional to $\tan(\gamma + \phi)$ instead of $\tan\gamma$ in the ideal case. The worm-driving efficiency and heat rejection estimate are

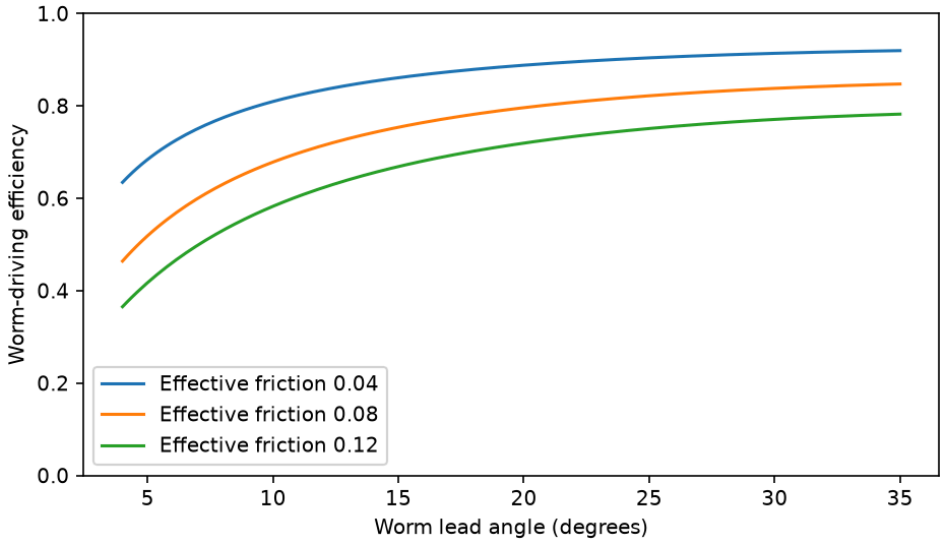
$$\eta = \frac{\tan\gamma}{\tan(\gamma + \phi)},$$

$$\dot{Q}_{loss} \simeq (1 - \eta) P_{in}.$$

For $\gamma = 12^\circ$, $\mu_{eff} = 0.08$ and 2 kW input, efficiency is about 0.714 and heat production about 0.57 kW before bearing and seal losses. A 40-tooth wheel with a two-start worm has ratio 20. Efficiency depends on lubricant and operating temperature. A ratio alone gives no thermal rating.

Can the load backdrive the worm?

Within this ideal Coulomb model, reversed sliding is possible when $\gamma > \phi$. A static friction estimate alone is inadequate as a holding specification when vibration, lubrication, wear or operating direction change.



Efficiency rises with lead angle over this range. Each curve assumes a constant effective friction coefficient.

Interactive calculation · Change lead angle, friction and heat loss

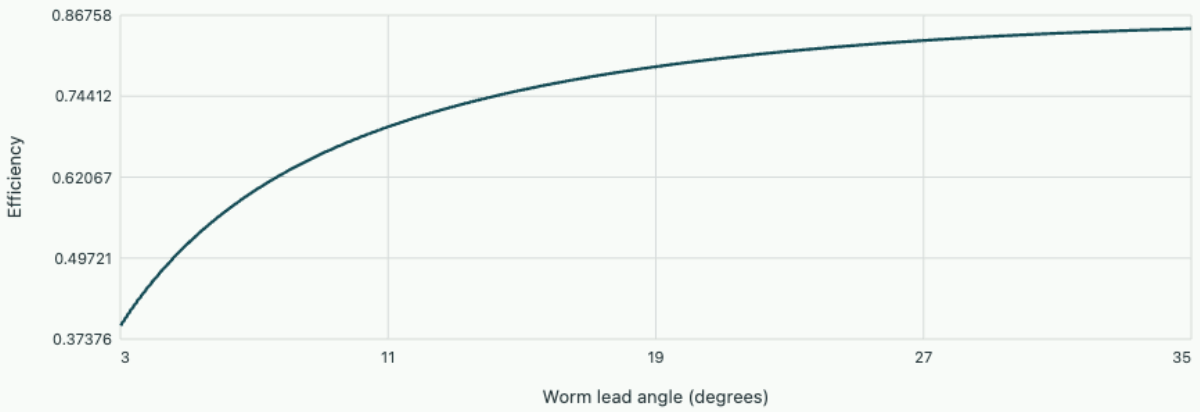
Raise friction while holding transmitted geometry fixed. The input power changes thermal loss but not the dimensionless efficiency in this idealised model.

Open this live calculation

Try the calculation

Effective friction coefficient **0.08**

Input power (W) **1000**



Driving worm

At15° lead angle efficiency=0.754, heat loss=246.4 W. Reverse self-locking in this ideal friction model requires lead below 4.574°.

Exercises on geometry and heat

A bevel pair has equal numbers of teeth. Which separating components on either shaft become equal?

▼ **Worked solution**

Both pitch cones have 45° half-angle, so the axial and radial components are each $F_t \tan\alpha/\sqrt{2}$. The tangential component still carries all transmitted torque.

The worm housing can reject 0.40 kW at the allowable temperature. What continuous input power fits the example model?

▼ **Worked solution**

Divide rejection by the loss fraction. With $\eta \simeq 0.714$, $P_{in} \simeq 1.40$ kW. An operating-temperature efficiency map and other losses are needed before specifying the drive.

Further reading · Budynas and Nisbett, Shigley's Mechanical Engineering Design, 10th ed., Chapter 15.

Design and analysis of machine components · 9. Can a mating pin enter every accepted hole?

A rectangular coordinate tolerance and a circular position tolerance accept different parts. Carry a hole requirement from its datum reference frame through its material condition to an actual inspection decision.

Construct the measurement before assigning error

A primary planar datum fitted by the specified simulator fixes normal translation and two tilts. A secondary side plane arrests one lateral translation and rotation about the primary normal. A tertiary end stop removes the final sliding freedom. Basic coordinates specify the intended hole axis in this frame. Datum order matters when the real surfaces are imperfect. A three-point seating scheme has different sensitivity to dirt or local form than a least-squares plane.

For a hole perpendicular to the primary plane in a thin plate, suppose tilt has been independently shown negligible. If the fitted centre departs from its basic coordinates by $\Delta x, \Delta y$, the smallest circular zone centred on the basic axis that contains it has diameter p. In a deep hole the entire derived axis must remain in the specified cylindrical zone, so a single face reading is insufficient.

$$p = 2\sqrt{\Delta x^2 + \Delta y^2}.$$

Why a larger internal feature can gain position tolerance

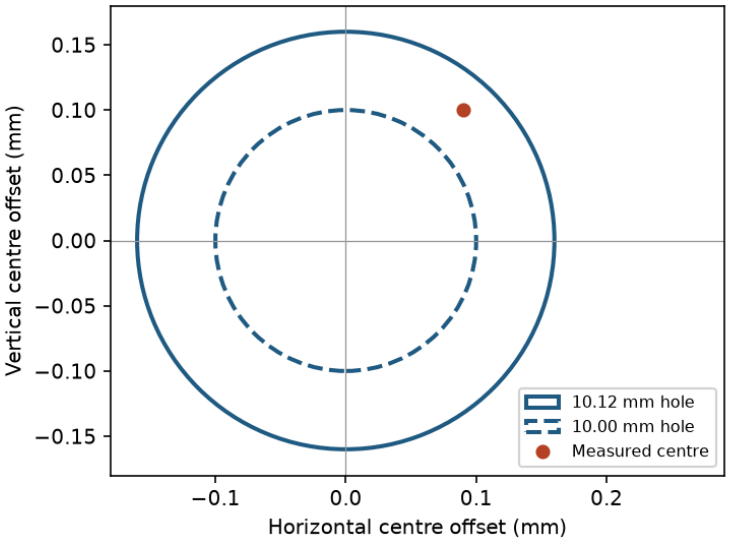
Suppose a mating design uses a hole between 10.00 and 10.20 mm and specifies a 0.20 mm diametral position tolerance at maximum material condition relative to fixed datums. For an internal feature maximum material is the smallest permissible diameter D_{MMC}. Holding a fixed virtual-condition boundary at the basic location gives

$$D_{VC} = D_{MMC} - t_{MMC} = 9.80 \text{ mm},$$
$$t_{allowed} = t_{MMC} + D_{actual} - D_{MMC}.$$

A measured 10.12 mm hole gains 0.12 mm diametral allowance. For offsets (0.09,0.10) mm its measured position is 0.269 mm, within the 0.32 mm allowance. An otherwise identical 10.00 mm hole fails the 0.20 mm position requirement. The smaller hole still satisfies its size limit. Under a rule without the material-condition modifier, the position allowance in this example stays 0.20 mm for both.

What a functional gauge actually checks

A suitably located 9.80 mm pin simulates the worst-case internal boundary only with the specified datum simulators engaged. Passing that pin alone does not check the hole’s upper size limit or all separate requirements on shape. The example assumes ideal form unless a test explicitly evaluates form.



The same measured centre lies inside the enlarged zone of a 10.12 mm hole and outside the zone of a hole at maximum material.

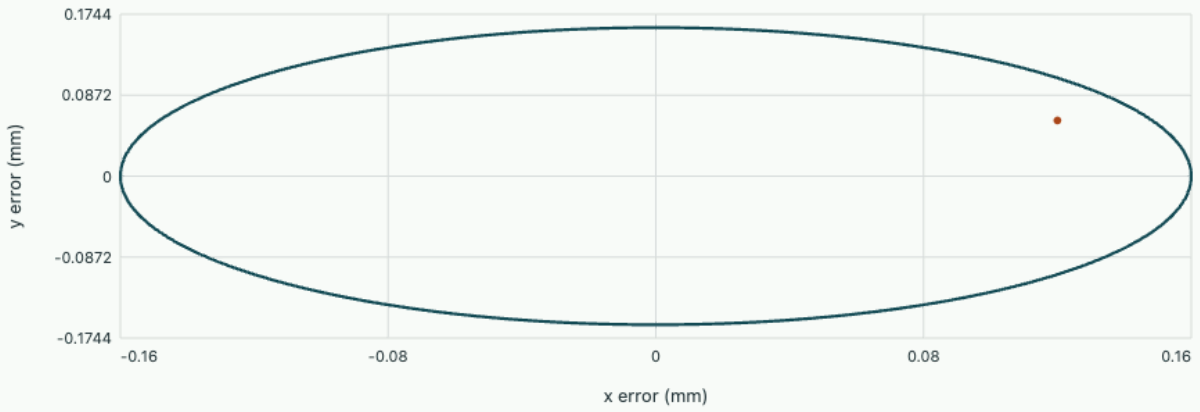
Interactive calculation · Move the measured centre within a material-dependent zone

Increase the measured hole diameter and inspect the allowed centre zone. Move the two measured offsets independently; acceptance uses their vector magnitude rather than separate coordinate limits.

Open this live calculation

Try the calculation

Actual hole diameter (mm)	**10.12**
Measured x error (mm)	**0.12**
Measured y error (mm)	**0.06**



Allowed centre zone Measured centre

Diametral position error 0.268 mm; allowed 0.320 mm: accepted. Virtual condition stays9.80 mm under the stated maximum-material rule.

Exercises on inspection decisions

A 10.08 mm hole has offsets (0.11,0.09) mm. Can the part be accepted on nominal readings?

▼ **Worked solution**

The diameter of its position zone is $2\sqrt{0.11^2 + 0.09^2} = 0.2843$ mm. The allowance is 0.28 mm. It narrowly fails on the stated values. With uncertainty comparable to 0.0043 mm, apply the drawing and inspection decision rule rather than rounding to two decimals before comparison.

A 30 mm long hole with nominal entrance centre is tilted by 0.3°. Where should a second measurement be taken?

▼ **Worked solution**

At the opposite end the centre departs by $30 \tan(0.3^\circ) = 0.157$ mm. The corresponding diametral deviation is about 0.314 mm if the axis is constrained to its basic orientation. Two widely separated centre measurements reveal tilt that an entrance-only check cannot see.

Further reading · Budynas and Nisbett, Shigley’s Mechanical Engineering Design, 10th ed., Chapter 20.